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The Bose-Einstein Condensation on Inhomogeneous Networks

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I. ABSTRACT

We present the new and unexpected results concerning the Bose–Einstein Condensation for the *Pure Hopping Model*, describing arrays of Josephson junctions located on non homogeneous networks. The amenable and the non amenable cases will be considered. The amenable case is summarized in the following table.

	ρ_c	R/T	d_G	d_{PF}	BEC	ρ -BEC
$\mathbb{Z}^d, d < 3$	∞	R	d	d	no	no
$\mathbb{Z}^d, d \geq 3$	$< \infty$	T	d	d	yes	yes
star graph	$< \infty$	R	1	0	no	no
$\mathbb{Z}^d \dashv \mathbb{Z}, d < 3$	$< \infty$	R	$d + 1$	d	no	no
$\mathbb{Z}^d \dashv \mathbb{Z}, d \geq 3$	$< \infty$	T	$d + 1$	d	yes	no
\mathbb{N}	∞	T	1	3	yes	no
$\mathbb{N} \dashv \mathbb{Z}$	$< \infty$	T	2	3	yes	no
$\mathbb{N} \dashv \mathbb{Z}^2$	$< \infty$	T	3	3	yes	yes

Here, $A \dashv B$ is the comb-shaped graph whose base-point is A , ρ_c is the critical density, R/T denotes the transience/recurrence of the adjacency, BEC (ρ -BEC) denotes the existence of locally normal states exhibiting BEC (exhibiting BEC at any finite mean density $\rho > \rho_c$).

The graphs under investigation are obtained by adding density zero perturbations to periodic amenable networks, and homogeneous Cayley Trees. The resulting (purely topological) model is described by a one particle Hamiltonian which is, up to an additive constant, the opposite of the adjacency operator on the graph. In the condensation regime, the particles condensate on the perturbed graph, even in the configuration space due to

non homogeneity. Roughly speaking, the system undergoes a sort of "dimension transition". We show for both amenable and non amenable situations, that it is enough to perturb in a negligible way the original graph in order to obtain a new network whose mathematical and physical properties dramatically change. The appearance of the *Hidden Spectrum* near the zero of the Hamiltonian, or equivalently below the norm of the adjacency. The latter is related to the value of the critical density and then with the appearance of the condensation phenomena. The investigation of the *recurrence/transience character* of the adjacency, which is connected to the possibility to construct locally normal states exhibiting the Bose–Einstein condensation. Finally, the study of the *volume growth of the wave function* of the ground state of the Hamiltonian, which is nothing but the generalized Perron–Frobenius eigenvector of the adjacency. This Perron–Frobenius weight describes the spatial distribution of the condensate and its shape is connected with the possibility to construct locally normal states exhibiting the Bose–Einstein condensation at a fixed density greater than the critical one.